Not only computing -

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Escher revisited

Every two or three years since the early 70s I have tried to write a program to create Escher like interlocking tiles of which Figure 1 is an example. I usually don't get very far in this endeavour. There are a number of reasons for this. The foremost of these is that I run out of spare time and some other more pressing programming task takes precedence. And we all know the problem of taking up a partly finished program some time after we have left it. More often, though, I find that the interactive method I have adopted to create the tiles is insufficiently general so that the whole program soon gets into a tangled mess of special cases. When this happens I simply lose heart and abandon the attempt.

Thus I have a notebook full of false starts and blind alleys as well as many tapes and discs crammed with programs in various stages of completeness in all sorts of languages for all sorts of computers (most of which can now be found only in museums).

The interesting thing about the problem is that there are only a limited number of possible ways of creating the interlocking forms. Twenty eight to be exact. It is fairly easy to write a long, messy program to cover all twenty eight possibilities. The mathematics for the arrangements had been sorted out by a German mathematics student, Heinrich Heesch, in 1932 although his tech niques were not published in an easily accessible form until 1963. Heesch's procedures were well formulated for computing in a paper by William Chow in 1979 *(Computer Graphics and Image Processing 9,* pp333 353). Jim McGregor and Alan Watt also had something useful to contribute in their book, *The Art of Micro computer Graphics* (Addison Wesley 1984). The difficulty is to write a nice, elegant interactive program which makes it easy for anyone to create any of the possible forms

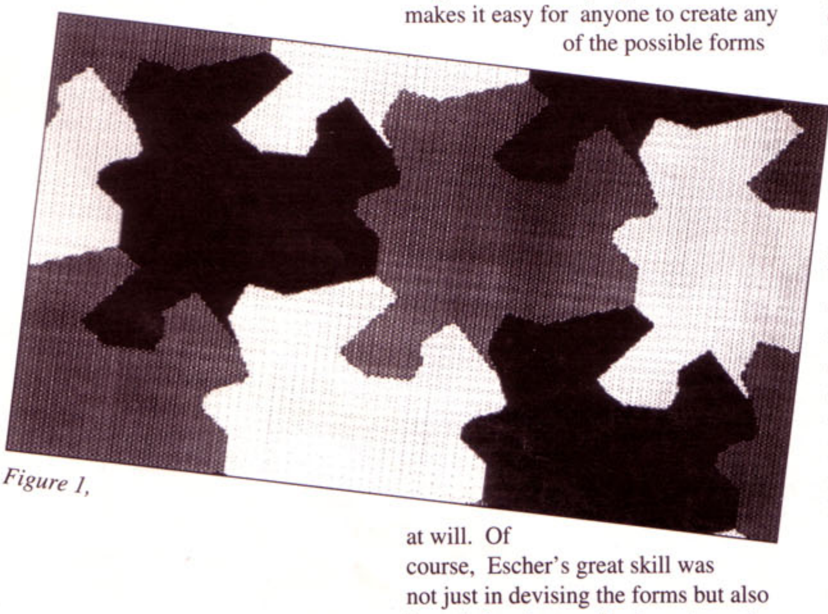


Figure 1,

at will. Of course, Escher's great skill was not just in devising the forms but also in embuing them with character and this is something that might not ever be possible to do by program.

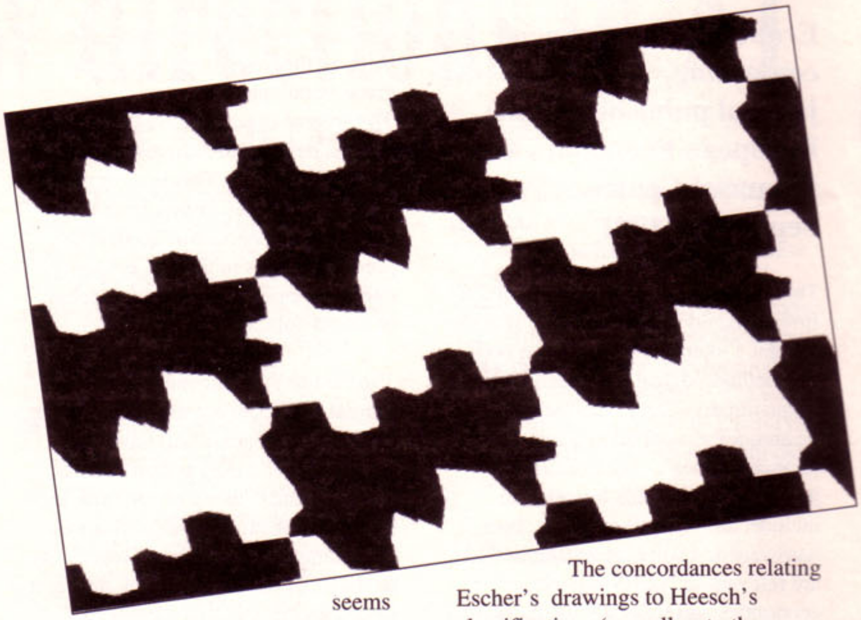
The reason that I have returned again to the problem is

Fgure 2

that I have recently acquired a new book on Maurits Escher's notebooks and drawings by an American researcher, Doris Schattschneider. Her excellent book is called V*isions of Symmetry* (Freeman and Co, 1990) and is an essential publication for anyone who wishes to understand Escher's methodswhich were clearly very ordered and painstaking. Schattschneider tells us that Escher (1898 1972) started on the interlocking forms in the 1930s, but that he did not know about Heesch's pioneering work until the early 60s. By then, he had derived almost all of the 28 forms himself. He did not use all of them in his drawings partly because of special restrictions on colouring and mirroring that he imposed on his forms. Naturally enough, too, he used some forms more often than otherspresumably because these had more 'character potential'. However, Escher's eldest son, George, is quoted in the book as saying that his father could recognise animal shapes in all sorts of random images such as clouds and wood grain and that he would decorate the walls of some of his rooms with figures and faces suggested by random splashes

also art

Figure 3



of paint. Incidentally, at the recent Fractals and Chaos II conference given by the BCS Computer Graphics and Displays Group, Richard Voss of IBM mentioned in passing research that he and colleagues had done on this phenomenon of being able to see recognisable shapes in essentially random patterns. He found that the phenomenon appears when the fractal dimensions of the pattern lies in a limited range and does not happen if the fractal dimen sion lies outside this. Something we will never now know is whether Escher's ability went beyond this apparent perceptual limit.

Heesch's classification

Heesch showed that interlocking forms could be made from edges arising from combinations of translation T, glide reflection G, and four types of rotation: a half turn C, a third turn C3, a quarter turn C4, and a one sixth turn C6. All these, except C, occur in pairs so that, for example, every translated or glide reflected edge has its twin somewhere else on the form. A C edge rotates around its midpoint. Because some types of edge cannot combine with others, it is a comparatively simple matter to show that there are only 28 possibilities. Heesch then classifies the forms by the operations that are necessary to create them listed in the order that they occur around the boundary. Thus TCCTCC a form that Escher seems to have been quite fond ofrequires a pair of translated lines and two pairs of lines which rotate about their centre points. For some reason the form TCTGG, which seems to me to be interesting, was not used by Escher at all.

Probably the simplest form, TTTT, is made up of just two pairs of translations. In this the left hand vertical edge has its identical twin on the right hand, and the top edge has its iden tical twin on the bottom (Figure 2). Another simple form is CCCC, where each edge is rotated about its midpoint (Figure 3).This form, by the way, shows something thaton the face of it is difficult to believe, namely that you can tile the plane with virtually any four sided convex or concave shape whatsoever! (Try it and see). This being so, it is surprising that tile and paving manufacturers aren't more imaginative with the shapes they give us.

Most forms require only two colours to distinguish them. Others, of which Figure 1 (the form TTTTTT) is an example, require three or more colours. Escher, of course, didn't only use different colours but added variation and interest to his drawings by giving different motifs to the same formsmaking a fish, say, in one direction and a bird in the other. Indeed most of his drawings use two or even three motifs. One drawing, shown on page 173 of Schattschneider's book, is a tour de force having no less than 12 different bird motifs!

Anyone interested in the methods and motivation of creative people in art and mathematics should get a copy of Visions of Symmetry. As modern books go it's not particularly expensive at around £28.00 especially as it's beautifully produced and exceptionally comprehensive.

The concordances relating Escher's drawings to Heesch's classifications (as well as to the Isohedral types set out by Grunbaum and Shephard and to the standard symme try groups) alone are worth the money and are essential tools for those trying to analyse the drawings either for artistic or programming purposes. Dr Schattschneider has done us all a tremendous service with her thorough, sensitive and insightful study of Escher's wartime notebooks and interlocking form draw ings and I thoroughly recommend the work to you. I know I'll be consulting it for years to come.

Round up

My thanks to all those who wrote me last year commenting on aspects of things I'd mentioned in these columns. I don't always get round to answering your notes individually and I apologise for this. I can only plead pressure of work. Anyway, many thanks for your interest. Please keep sending in your letters. Quite a few of you had encountered stereo lithography, a subject I touched on one or two issues ago. It appears that the use of computer controlled lasers solidifying vats of polymer plastic is rather more widespread than I thought. I am arranging to see a stereo lithographic system soon and although this is probably too expensive for an educational establishment to buy, I am hoping that I can persuade someone to let my Master's students use one for their sculpture projects. I will keep you informed on progress.